nag_deviates_gamma_dist (g01ffc)

1. Purpose

nag_deviates_gamma_dist (g01ffc) returns the deviate associated with the given lower tail probability of the gamma distribution.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_deviates_gamma_dist(double p, double a, double b, double tol,
       NagError *fail)
```

3. Description

The deviate, g_p, associated with the lower tail probability, p, of the gamma distribution with shape parameter α and scale parameter β, is defined as the solution to

$$P(G \leq g_p : \alpha, \beta) = p = \frac{1}{\beta^\alpha \Gamma(\alpha)} \int_0^{g_p} e^{-G/\beta} G^{\alpha-1} \, dG \qquad 0 \leq g_p < \infty;\ \alpha, \beta > 0.$$

The method used is described by Best and Roberts (1975) making use of the relationship between the gamma distribution and the χ^2 distribution.

Let $y = 2g_p/\beta$. The required y is found from the Taylor series expansion

$$y = y_0 + \sum_r \frac{C_r(y_0)}{r!} \left(\frac{E}{\phi(y_0)} \right)^r$$

where y_0 is a starting approximation

$$\begin{array}{rcl} C_1(u) & = & 1 \\ C_{r+1}(u) & = & \left(r\Psi + \frac{d}{du}\right) C_r(u) \\ \Psi & = & \frac{1}{2} - \frac{\alpha-1}{u} \\ E & = & p - \int_0^{y_0} \phi(u) \, du \\ \phi(u) & = & \frac{1}{2^\alpha \Gamma(\alpha)} e^{-u/2} u^{\alpha-1} \end{array}.$$

For most values of p and α the starting value

$$y_{01} = 2\alpha \left(z \sqrt{\frac{1}{9\alpha}} + 1 - \frac{1}{9\alpha} \right)^3$$

is used, where z is the deviate associated with a lower tail probability of p for the standard Normal distribution.

For p close to zero,

$$y_{02} = \left(p\alpha 2^\alpha \Gamma(\alpha)\right)^{1/\alpha}$$

is used.

For large p values, when $y_{01} > 4.4\alpha + 6.0$

$$y_{03} = -2 \left(\ln(1-p) - (\alpha - 1) \ln(\tfrac{1}{2} y_{01}) + \ln(\Gamma(\alpha)) \right)$$

is found to be a better starting value than y_{01}.

For $\alpha \leq 0.16, p$ is expressed in terms of an approximation to the exponential integral and y_{04} is found by Newton–Raphson iterations.

Seven terms of the Taylor series are used to refine the starting approximation, repeating the process if necessary until the required accuracy is obtained.

4. Parameters

p

Input: the probability, p, from the required gamma distribution.
Constraint: $0.0 \leq \mathbf{p} < 1.0$.

a

Input: the shape parameter, α, of the gamma distribution.
Constraint: $0.0 < \mathbf{a} \leq 10^6$.

b

Input: the scale parameter, β, of the gamma distribution.
Constraint: $\mathbf{b} > 0.0$.

tol

Input: the relative accuracy required by the user in the results. The smallest recommended value is $50 \times \delta$, where $\delta = \max(10^{-18},$ ***machine precision***$)$. If nag_deviates_gamma_dist is entered with **tol** less than $50 \times \delta$ or greater than or equal to 1.0, then $50 \times \delta$ is used instead.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

On any of the error conditions listed below except **NE_ALG_NOT_CONV** nag_deviates_gamma_dist returns 0.0.

NE_REAL_ARG_LT

On entry, **p** must not be less than 0.0: $\mathbf{p} = \langle value \rangle$.

NE_REAL_ARG_GE

On entry, **p** must not be greater than or equal to 1.0: $\mathbf{p} = \langle value \rangle$.

NE_REAL_ARG_LE

On entry, **a** must not be less than or equal to 0.0: $\mathbf{a} = \langle value \rangle$.
On entry, **b** must not be less than or equal to 0.0: $\mathbf{b} = \langle value \rangle$.

NE_REAL_ARG_GT

On entry, **a** must not be greater than 10^6: $\mathbf{a} = \langle value \rangle$.

NE_PROBAB_CLOSE_TO_TAIL

The probability is too close to 0.0 for given **a** to enable the result to be calculated.

NE_ALG_NOT_CONV

The algorithm has failed to converge in 100 iterations.
A larger value of **tol** should be tried. The result may be a reasonable approximation.

NE_GAM_NOT_CONV

The series used to calculate the gamma probabilities has failed to converge.
This is an unlikely error exit.

6. Further Comments

6.1. Accuracy

In most cases the relative accuracy of the results should be as specified by **tol**. However for very small values of α or very small values of p there may be some loss of accuracy.

6.2. References

Best D J and Roberts D E (1975) The percentage points of the χ^2 distribution *Appl. Stat.* **24** Algorithm AS91 385–388.

7. See Also

None.

8. Example

Lower tail probabilities are read for several gamma distributions, and the corresponding deviates calculated and printed, until the end of data is reached.

8.1. Program Text

```
/* nag_deviates_gamma_dist(g01ffc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 1, 1990.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg01.h>

main()
{
  double a ,b, p, g;
  double tol = 0.0;
  static NagError fail;

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("g01ffc Example Program Results\n");
  Vprintf("    p       a       b           g\n\n");
  while (scanf("%lf %lf %lf", &p, &a, &b) != EOF)
    {
      g = g01ffc(p, a, b, tol, &fail);
      if (fail.code==NE_NOERROR)
        Vprintf("%8.3f%8.3f%8.3f%10.3f\n", p, a, b, g);
      else
        Vprintf("%8.3f%8.3f%8.3f%10.3f\n Note: %s\n", p, a, b, g,
                fail.message);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01ffc Example Program Data
 0.0100   1.0 20.0
 0.4279   7.5  0.1
 0.8694  45.0 10.0
```

8.3. Program Results

```
g01ffc Example Program Results
    p       a       b           g

   0.010   1.000  20.000     0.201
   0.428   7.500   0.100     0.670
   0.869  45.000  10.000   525.979
```